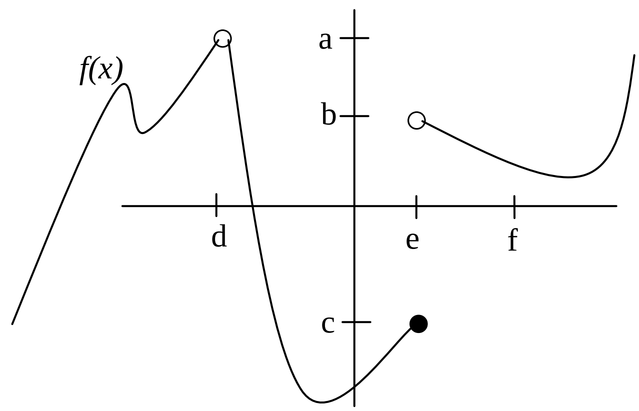
KCATM 2011
Calculus Test - **NO Calculator**

1) Given the graph of *f*, $\lim_{x \to e^+} f(x) =$



a) a

b) b

c) c

d) d

e) e

2) Given: $f(x) = (kx+2)^4$, where *k* is a constant. If the slope of the tangent line at x = 0 is 8, then what is the value of *k*?

a) $\frac{3}{4}$

b) $\frac{1}{4}$

c) $\frac{3}{8}$

d) $\frac{7}{8}$

e) $\frac{2}{5}$

3) Find $\lim_{x\to 0} \frac{3(\sin x)(\cos x)}{2x} =$

a) 0

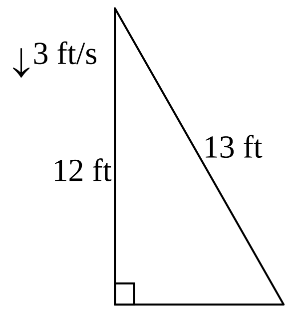
b) $\frac{1}{2}$

c) 1

d) $\frac{3}{2}$

e) Does Not Exist

4) A 13 foot ladder is leaning against the wall of a building. The top of the ladder touches the building at a point 12 feet above the ground *(see figure)*. The top of the ladder is moving down the side of the building at a rate of 3 feet per second. How fast is the base of the ladder being pulled away?



5) At what value(s) of x does $f(x) = \frac{x^4}{4} - x^3 + x^2 + 3$ have a relative maximum?

a) 0 and 2 only

b) 0 and 1 only

c) 1 only

d) 1 and 2 only

e) 0, 1, and 2

6) $\int_0^{\pi/4} (\sec^2 x)(\tan^2 x)dx$

a) $-1/2$

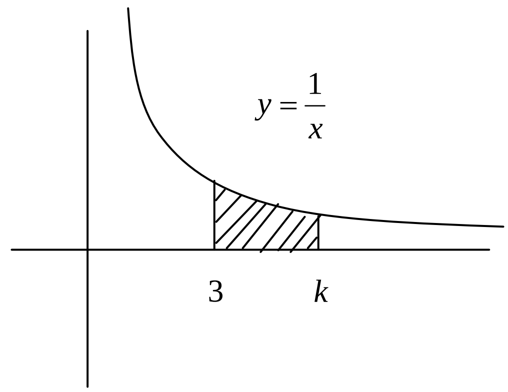
b) $-1/3$

c) $1/5$

d) $1/3$

e) $1/2$

7) For the Figure, the area of the shaded region is $\ln 9$ when k is



a) e

b) e^2

c) 4

d) 9

e) 27

8) Which of the following integrals gives the volume of the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$, rotated around the line $x = 4$.

a) $\pi \int_0^2 (4 - \sqrt{x})^2 \, dx$

b) $\pi \int_0^4 (4 - \sqrt{y})^2 \, dy$

c) $\pi \int_0^4 (\sqrt{x})^2 \, dx$

d) $\pi \int_0^2 (\sqrt{x} - 4)^2 \, dx$

e) $\pi \int_0^2 (4 - y^2)^2 \, dy$

9) Given $f(x) = x^2 + 1$, $h(x) = 4x - 3$, *g(x)* is a continuous function on the open interval *(0,5)* and $h(x) \le g(x) \le f(x)$ on the same interval. Find $\lim_{x \to 2} g(x) =$

a) 2

b) 3

c) 4

d) 5

e) Cannot be determined

10) Find $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}$ dx

a) $\ln\left| e^x - \frac{1}{e^x} \right| + C$

b) $\ln\left| e^x + \frac{1}{e^x} \right| + C$

c) $\ln\left| e^x - 1 \right| + C$

d) $\ln\left| \frac{1}{e^x} \right| + C$

f) $\ln\left| e^{-x} - 1 \right| + C$

11) The graph of the derivative of a twice-differentiable function f is shown below. If $f(1) = -2$, which of the following is true?

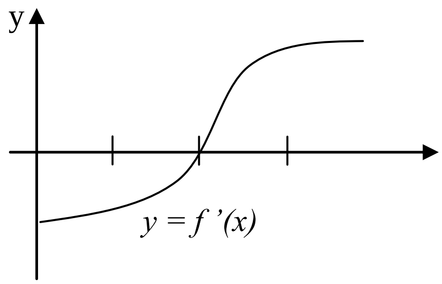
a) $f'(2) < f''(2) < f(2)$

b) $f''(2) < f'(2) < f(2)$

c) $f'(2) < f(2) < f''(2)$

d) $f(2) < f'(2) < f''(2)$

e) $f(2) < f'(2) < f''(2)$



12) The tangent line to the graph $y = e^{2-x}$ at the point $(1, e)$ intersects both coordinate axes. What is the area of the triangle formed by this tangent line and the coordinate axis?

a) $\frac{e}{2}$

b) e

c) $\frac{3e}{2}$

d) $2e$

e) $\frac{5e}{2}$

13) $\int_{3}^{\infty} \frac{1}{x^2} dx =$

a) $\frac{1}{3}$

b) $\frac{2}{3}$

c) 3

d) 4

e) Divergent

14) $\int x^2 \mathrm{e}^{\mathrm{x}} \; dx =$

a) $2\mathrm{e}^{\mathrm{x}} + C$

b) $x^2\mathrm{e}^{\mathrm{x}} - 2x\mathrm{e}^{\mathrm{x}} + 2\mathrm{e}^{\mathrm{x}} + C$

c) $\mathrm{x}^2\mathrm{e}^{\mathrm{x}} + 2xe^{x} + C$

d) $\frac{x^3\mathrm{e}^{\mathrm{x}}}{3} + C$

e) $\frac{\mathrm{x}^3\mathrm{e}^{\mathrm{x}}}{3} + x^2 e^{x} + C$

15) A particle moves in the *xy*-plane so that at any time *t* its coordinates are $x = 3t^2 + t - 4$ and $y = \sin(3t)$. At $t = \pi/6$, its acceleration vector is

a) $\langle 6, -1 \rangle$

b) $\langle 6, 1 \rangle$

c) $\langle 6, -9 \rangle$

d) $\langle \pi, -5 \rangle$

e) $\langle 6, 9 \rangle$

16) If $S_n = \left(\frac{(n+2)^{81}}{3^n} \right) \left(\frac{3^{n+3}}{\left(42^{789} + n^3\right)^{27}} \right)$, to what number does the sequence $\{S_n\}$ converge?

a) 0

b) 3

c) 9

d) 27

e) ∞

17) What are the first four nonzero terms in the power series expansion of e^{3x} about x = 0?

a) $x + 3x^2 + \frac{9x^3}{2} + \frac{9x^4}{2}$

b) $1 + 3x + \frac{9x^2}{2} + \frac{9x^3}{2}$

c) $\frac{1}{3} + x + \frac{3x^2}{2} + \frac{3x^3}{2}$

d) $1 - 3x + \frac{9x^2}{2} - \frac{9x^3}{2}$

e) $1 + 3e^x + 9e^x + 27e^x$

18) $\lim_{h\to\pi} \frac{1}{h-\pi} \int_{\pi}^{h} \frac{\cos^2 t}{t^2}\ dt$

a) ∞

b) $\frac{1}{\pi^2}$

c) 0

d) 1

e) $\frac{\pi}{2}$

19) Given: $f(x, y) = 4xy - x^2 - y^3$, find the sum of the slopes of the surface defined by $f(x, y)$ in the x- and y- directions at the point (1, 1, 2).

a) 0

b) 1

c) 2

d) 3

e) 4

20) Find the equation of the tangent plane to the hyperboloid given by the equation $z^2 - 2x^2 - 3y^2 = 4$, at the point (1, -1, 3)

a) $-2x + 3y + 3z = 4$

b) $-3x - 2y + 4z = 4$

c) $2x - 3y + z = 4$

d) $-4x - 3y + z = 4$

e) $4x + y + 2z = 4$