

Suppose  $f(x)$  is a continuous, twice-differentiable function. Values of  $x$ ,  $f(x)$  &  $f'(x)$  are provided in the table below. You will use this table to answer questions 1-5.

$x$	0	1	2	3	5
$f(x)$	1	3	5	7	2
$f'(x)$	-5	1	-7	-2	3

1) What is the average rate of change of  $f(x)$  over the interval  $[0, 5]$ ?

- A) 0  
B)  $1/5$   
C) 3  
D) 5  
E) 10

2) Write the equation of the line tangent to  $f(x)$  at  $x = 2$ .

- A)  $y = 5x - 17$   
B)  $y = 5x - 3$   
C)  $y = -7x - 9$   
D)  $y = -7x + 9$   
E)  $y = -7x + 19$

3) Evaluate  $\frac{d}{dx}(f(x))^2$  at  $x = 3$ .

- A) -28  
B) -14  
C) 14  
D) 28  
E) 49

4) Use a trapezoidal sum with four subintervals to estimate  $\int_0^5 f(x) dx$ .

- A) 0  
B) 7  
C) 14  
D) 21  
E) 28

5) Evaluate  $\int_0^5 f'(x) dx$ .

- A) 1  
B) 2  
C) 3  
D) 5  
E) 7

For questions 6-9, suppose  $f(x) = \frac{x-2}{x^2-4}$ .

6) Compute  $\lim_{x \rightarrow 2} f(x)$ .

A) 0

B) 1/4

C) 1/2

D) 1

E) The limit does not exist.

7) Compute  $f'(4)$ .

A) -4

B) -1/2

C) -1/36

D) 1/36

E) 1/2

8) Compute  $\int_5^7 f(x) dx$ .

A)  $\ln\left(\frac{5}{3}\right)$

B)  $\ln\left(\frac{7}{5}\right)$

C)  $\ln\left(\frac{9}{7}\right)$

D)  $\ln\left(\frac{11}{9}\right)$

E)  $\ln\left(\frac{13}{11}\right)$

9) Compute  $\lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$ .

A) -1

B) -1/49

C) -1/25

D) 1/25

E) 1/49

10) Compute  $\int x\sqrt{25+x} dx$ .

A)  $x^2(25+x)^{1/2} + C$

B)  $\frac{x^2}{3}(25+x)^{3/2} + C$

C)  $\frac{1}{2\sqrt{x^2+25x}} + C$

D)  $\frac{2}{5}(x+25)^{5/2} - \frac{50}{3}(x+25)^{3/2} + C$

E)  $\frac{2}{3}(x+25)^{3/2} - 50(x+25)^{1/2} + C$

11) Compute  $\int x\sqrt{25+x^2} dx$ .

A)  $\frac{1}{3}(25+x^2)^{3/2} + C$

B)  $\frac{2}{3}(25+x^2)^{3/2} + C$

C)  $\frac{4}{3}(25+x^2)^{1/2} + C$

D)  $\frac{1}{4\sqrt{25+x^2}} + C$

E)  $\frac{1}{2\sqrt{25+x^2}} + C$

12) Which of the following integrals is equivalent to  $\int \sqrt{25-x^2} dx$  if the substitution  $x = 5 \sin \theta$  is used?

A)  $\int 5 \cos \theta d\theta$

B)  $\int 5 \sin \theta d\theta$

C)  $\int 25 \sin^2 \theta d\theta$

D)  $\int 25 \cos^2 \theta d\theta$

E)  $\int \cos^2 \theta d\theta$

13) Compute  $\frac{d}{dx} \int_0^x \sqrt{25-t^2} dt$ .

A)  $\sqrt{25-x^2}$

B) 0

C)  $\frac{-x}{\sqrt{25-x^2}}$

D)  $\frac{x}{\sqrt{25-x^2}}$

E)  $\frac{2}{3}(25-x^2)^{3/2}$

14) Compute  $\frac{d}{dx} e^{\tan x}$ .

A)  $e^{\tan x}$

B)  $e^{\tan x} \csc^2 x$

C)  $e^{\tan x} \sec^2 x$

D)  $-e^{\tan x} \sec^2 x$

E)  $-e^{\tan x} \csc^2 x$

15) Compute  $\int \frac{1}{x^2-4} dx$ .

A)  $\frac{1}{4} \ln(x^2-4) + C$

B)  $\frac{1}{4} \ln|x^2-4| + C$

C)  $\frac{1}{4} \ln\left(\frac{x-2}{x+2}\right) + C$

D)  $\frac{1}{4} \ln\left|\frac{x-2}{x+2}\right| + C$

E)  $\frac{1}{4} \ln\left|\frac{x+2}{x-2}\right| + C$

For questions 16-19, suppose  $f(2) = 5$ .

16) Which of the following statements is/are true if  $f'(2) = 4$ ?

- I) The equation of the line tangent to  $f(x)$  at  $x = 2$  is  $y = 4x - 3$ .
- II)  $f(x)$  is differentiable at  $x = 2$ .
- III)  $f(x)$  is continuous at  $x = 2$ .

- A) I only
- B) II only
- C) III only
- D) I and III only
- E) I, II, and III

17) Which of the following statements is/are true if  $\lim_{x \rightarrow 2^-} f(x) = 3$  &  $\lim_{x \rightarrow 2^+} f(x) = 5$ ?

- A)  $f(x)$  is continuous at  $x = 2$ .
- B)  $f(x)$  is differentiable at  $x = 2$ .
- C)  $f(x)$  has a jump discontinuity at  $x = 2$ .
- D)  $f(x)$  has a removable discontinuity at  $x = 2$ .
- E)  $f(x)$  has an oscillating discontinuity at  $x = 2$ .

18) Which of the following statements is/are true if  $\lim_{x \rightarrow 2} f(x) = 6$ ?

- A)  $f(x)$  is continuous at  $x = 2$ .
- B)  $f(x)$  is differentiable at  $x = 2$ .
- C)  $f(x)$  has a jump discontinuity at  $x = 2$ .
- D)  $f(x)$  has a removable discontinuity at  $x = 2$ .
- E)  $f(x)$  has an oscillating discontinuity at  $x = 2$ .

19) Which of the following statements is/are true if  $\lim_{x \rightarrow 2} f(x) = 5$ ?

- A)  $f(x)$  is continuous at  $x = 2$ .
- B)  $f(x)$  is differentiable at  $x = 2$ .
- C)  $f(x)$  has a jump discontinuity at  $x = 2$ .
- D)  $f(x)$  has a removable discontinuity at  $x = 2$ .
- E)  $f(x)$  has an oscillating discontinuity at  $x = 2$ .

20) Find the area of the region between by the curves  $f(x) = \sqrt{x}$  &  $g(x) = x^2$ .

A)  $1/27$

B)  $1/15$

C)  $1/9$

D)  $1/6$

E)  $1/3$

21) Find the volume of the solid created when  $f(x) = \sqrt{x}, 0 \leq x \leq 4$ , is rotated about the  $x$ -axis.

A)  $\pi$

B)  $2\pi$

C)  $4\pi$

D)  $8\pi$

E)  $16\pi$

22) Evaluate  $\int_0^{\infty} xe^{-x^2} dx$ .

A)  $1/4$

B)  $1/2$

C)  $1$

C)  $2$

E)  $4$

23) Solve the initial value problem:  $\frac{dy}{dx} = xy; y(0) = 3$ .

A)  $y = e^{x^2/2} + C$

B)  $y = -e^{x^2/2} + C$

C)  $y = -3e^{x^2/2} + C$

D)  $y = 3e^{x^2/2} + C$

E)  $y = 3e^{x/2} + C$

24) Find the slope of the line tangent to  $(x + y)^2 = 3x$  at the point  $(3, 0)$ .

A)  $-1$

B)  $-1/2$

C)  $0$

D)  $1/2$

E)  $1$

For questions 25-28, suppose the velocity of a particle moving along a horizontal number line at time  $t$ , is given by the equation  $v(t) = \cos(e^t) + 2$ , for  $t \geq 0$ .

25) Find the acceleration of the particle at  $t = 1$ .

A) -1.117

B) -0.617

C) -0.117

D) 0.383

E) 0.883

26) Find the average velocity of the particle over the interval  $[0, 2]$ .

A) 0.386

B) 0.886

C) 1.386

D) 1.886

E) 2.386

27) Find the average acceleration of the particle over the interval  $[0, 2]$ .

A) -2.046

B) -1.546

C) -1.046

D) -0.546

E) -0.046

28) Suppose  $s(t)$  represents the position of the particle at time  $t$ . If  $s(0) = 3$ , find  $s(2)$ .

A) 3.772

B) 4.772

C) 5.772

D) 6.772

E) 7.772

## KCATM Calculus Answer Key 2014

1. B
2. E
3. A
4. D
5. A
6. B
7. C
8. C
9. C
10. D
11. A
12. D
13. A
14. C
15. D
16. E
17. C
18. D
19. A
20. E
21. D
22. B
23. D
24. B
25. A
26. D
27. E
28. D