2017 KCATM
High School Math Contest

Name: _____

Grade: _____

High School: _____

e-mail address: _____

phone number: _____

INSTRUCTIONS

The following test consists of 20 questions. Use whatever resources you like to solve these problems. Each question is worth 4 points. Partial credit will be given for making progress. However, you must show your work to get any credit.

PLEASE PRINT THIS OFF IN LANDSCAPE MODE!

THE CONTEST

There is no cost for this contest. 9th and 10th graders will be graded in one category, while 11th and 12th graders will be graded in another. The top 3 students in each category will be informed by March 31st, and recognized at the annual banquet on Monday, April 24th.

ALL ENTRIES MUST BE POST-MARKED NO LATER THAN MARCH 25TH.

MAIL YOUR ANSWERS

Mike Round
13234 Long Street
Overland Park, KS 66213

QUESTIONS

High School Test Coordinator: Mike Round
(913) 515-3911 round12345@aol.com

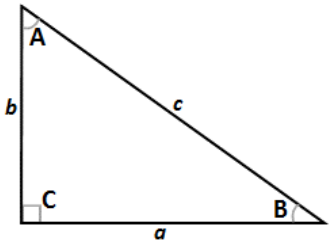
1. I'm selling 256 products, initially for $100 each. Each day, I sell half of the products available. The next day, I reduce the price 50%, and the process continues. If products cannot be divided into parts, and if all transactions are rounded to the nearest penny, then what the average price I received for all products sold – *to the penny*?

2. Office visits at a doctor's office occur with the following length (in minutes) and frequency. The average time per visit (x) was calculated, and 10 appointments were scheduled every x minutes, starting at 8:00 AM. The doctor goes on break immediately after treating the 10^{th} patient.

VISIT LENGTH *(minutes)*	FREQUENCY
5	40%
10	35%
15	20%
30	5%

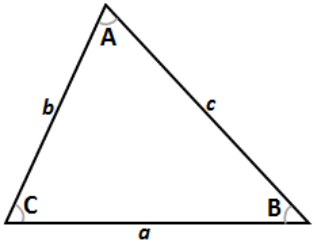
If patients show up exactly at their scheduled time, what is the difference (in minutes) between the earliest and latest times the doctor will go on break?

3. Prove the “Law of Cosines” formula.



Pythagorean Theorem

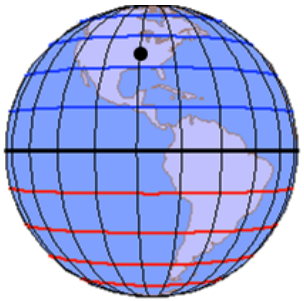
$$c^2 = a^2 + b^2$$



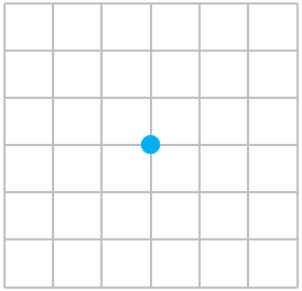
Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$$

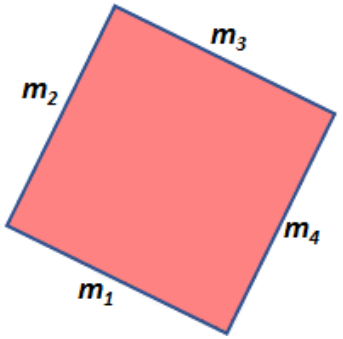
4. Assume the earth is an exact sphere, with diameter of 7,917.5 miles. Kansas City’s latitude is approximately 39˚ north of the equator. If the earth makes one complete rotation daily, how fast is a person in Kansas City spinning (in miles / hour)?



5. Starting in the middle of a grid, I move one unit, chosen randomly, left, right, up, or down. If I repeat this process two more steps, what are the odds I will end up where I started?



6. The slopes of the 4 lines in this square are noted. What is $m_1 \times m_2 \times m_3 \times m_4$?



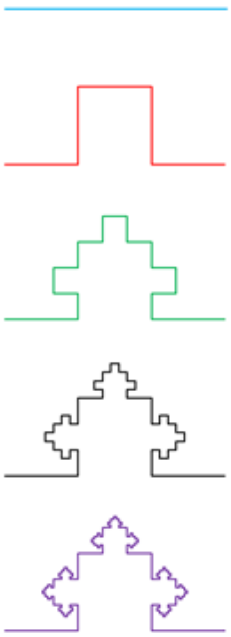
7. The length of a "true" year is approximately 365.2422 days. A leap year occurs every four years, except if the year is divisible by 100. Then, the year is *not* a leap year – unless the year is divisible by 400, in which case, the year *is* a leap year! Starting from 1/1/2000, how far ahead / behind is the calendar by 12/31/2999?

8. $f(x)$ has roots $x = 4$, $x = -4$, and $x = 2$, while $g(x) = -2x + 4$. At what coordinates does $f(x) = g(x)$?

9. At what temperature are Celsius and Fahrenheit equal?

Scale	Freezing Point	Boiling Point
Celsius	0°	100°
Fahrenheit	32°	212°

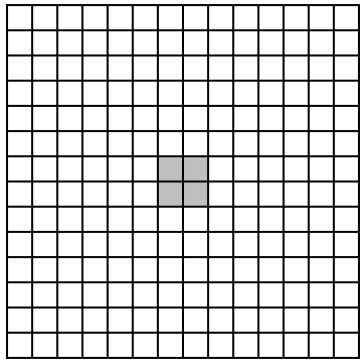
10. The length of the initial image is 100 units. Each iteration, a change takes place in the middle third of the shown segments. What is the length of the final image?



11. If I invest $100 at the beginning of the year at an annual interest rate of 6%, and this interest rate compounds continuously, how much money do I have at the end of the year?

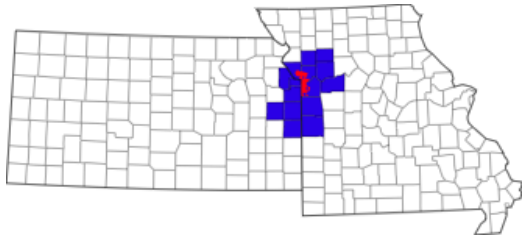
$$amount = 100\left(1+\frac{.06}{n}\right)^{n}$$

12. Assume each cell in this grid contains a tree. A fire starts in the middle of the forest. New cells catch fire when they are touching 2 or more cells that are on fire. "Touching" means being adjacent or diagonal to a cell on fire. Each minute, the grid is updated. After 5 minutes, what % of the forest is on fire?



13. I bought a 51¢ candy-bar with a dollar bill. Assuming there are quarters, dimes, nickels, and pennies in the register, how many ways can I receive proper change?

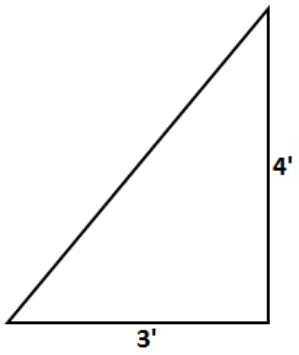
14. The grassland fires in SE Kansas have thus far burned 650,000 acres, making it the largest fire in the history of Kansas. If the Kansas City metro area (below) is 8,472 square miles, what % of this area would be burned in a similarly-sized fire?



15. What is the largest cube that will fit in a sphere with surface area of 100 cubic inches?

16. Suppose each set of your grandparents had four children (two boys and two girls), and each of those children married and had four children (two boys and two girls). If married girls took their husband's surname, what percent of your cousins have the same last name as you?

17. What is the area of the largest rectangle that will fit inside this triangle?



18. $i - i^{i^{i^{i}}} =$

19. $Pythagorean\ Win\ \% = \frac{RS^2}{RS^2 + RA^2}$ (based on runs) is used as an approximation for $Win\ \% = \frac{W}{W+L}$ (based on games). Which season below did the Royals win more games than they "should have" (using the difference between the two percentages)?

Year	Wins	Losses	Runs Scored	Runs Allowed
2016	81	81	675	712
2015	95	67	724	641
2014	89	73	651	624

20. Candidate 1 received two-thirds of the votes during the first one-third of the voting. Candidate 2 received three-fourths of the votes during the next one-fourth of the voting. What % of the remaining vote does Candidate 3 need to be declared the *only* winner?