Kansas City Area Teachers of Mathematics
2013 KCATM Contest

Mathletics

Grade 8

Instructions:

- Do **NOT** turn this page until instructed to do so.
- WRITE YOUR **TEAM NUMBER** AND **SCHOOL NAME** ON THE LINE PROVIDED ON THE FRONT OF EACH SHEET EACH TIME YOU BEGIN A NEW PROBLEM.
- You **may** use calculators on this test (*not* cell phone calculators).
- Blank scratch paper can be used. Do **NOT** write on the team number card.
- You may **not** use rulers, protractors or other measurement

Problem # 1

Team Number: _____ School: _____

Team Members: _____Gr ____

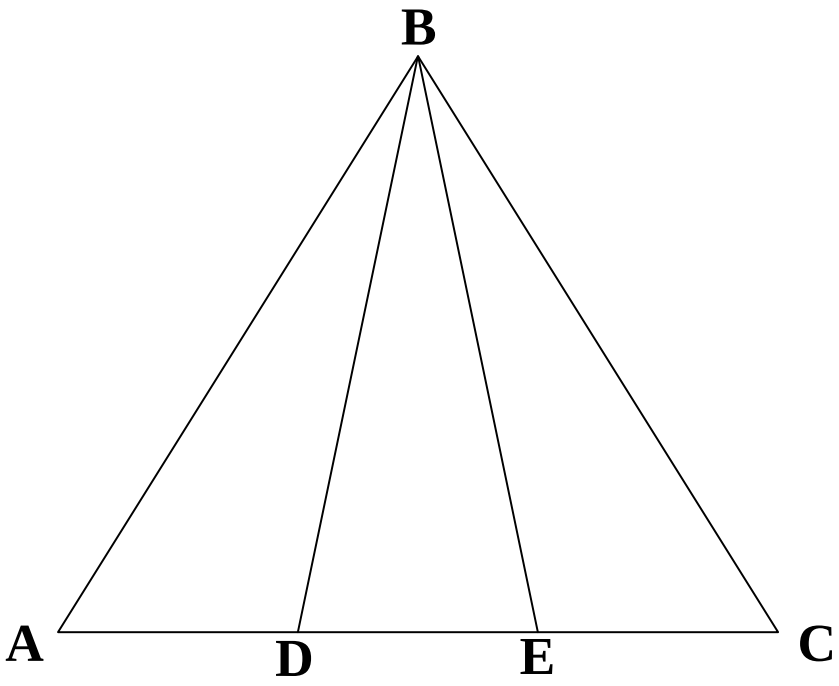
_____Gr ____

_____Gr ____

Problem 1 **2 points** **2 minutes**

Given: ABC is an equilateral triangle. D and E are points that divide the base ($\overline{AC}$) into three equal parts; AC = 3 cm

1. How many triangles are depicted in the diagram below?
2. What is the **difference** between the area of triangle ABD and the area of triangle DBC in terms of triangle ABC?



Answers: _____

_____

Team Number: _____ **School:** _____

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Problem # 2

Do NOT turn the page until you are told to do so.

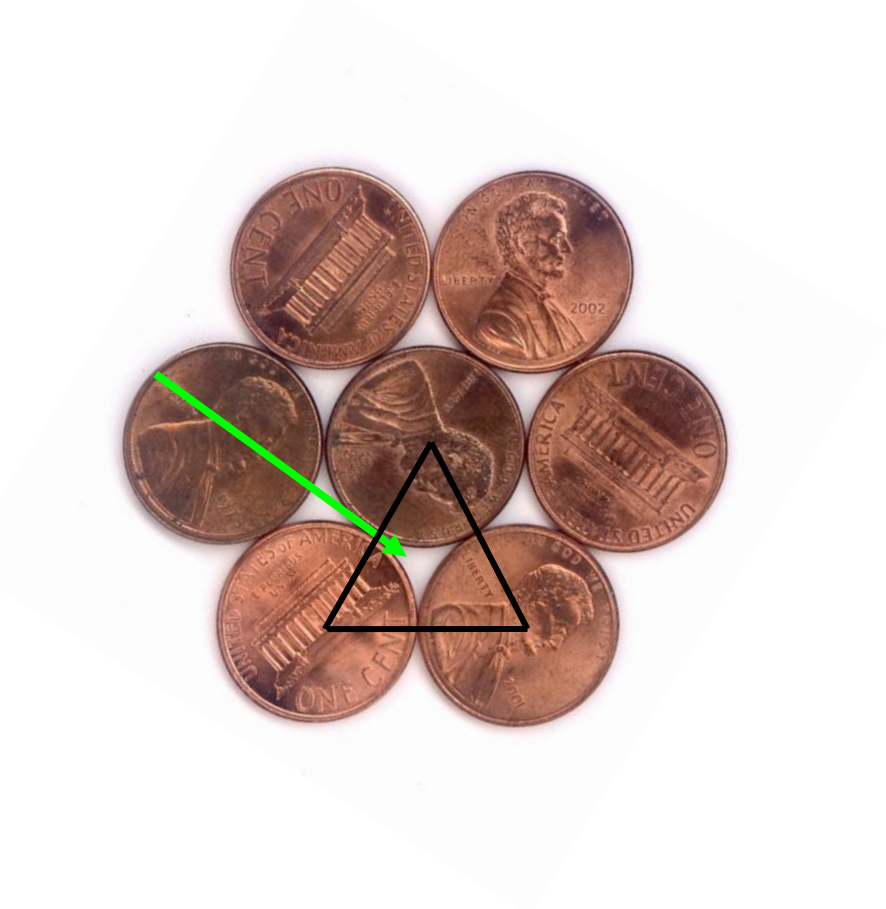
Team Number: _____ **School:** _____

Problem 2 **3 points** **3 minutes**

Modified from problem #46 at //www.eklhad.net/funmath.html

What is the area trapped between three pairwise tangent circles (of radius = 1 unit) in a plane?

For example, put three pennies on a table and arrange them in a triangle so that they touch each other. Pretend that the radius of the pennies is 1 unit. **Determine the area of the little three-pointed shape** made by the pennies (the three pairwise tangent circles)?



Answer: _____

Team Number: _____ School: _____

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Problem # 3

Do NOT turn the page until you are told to do so.

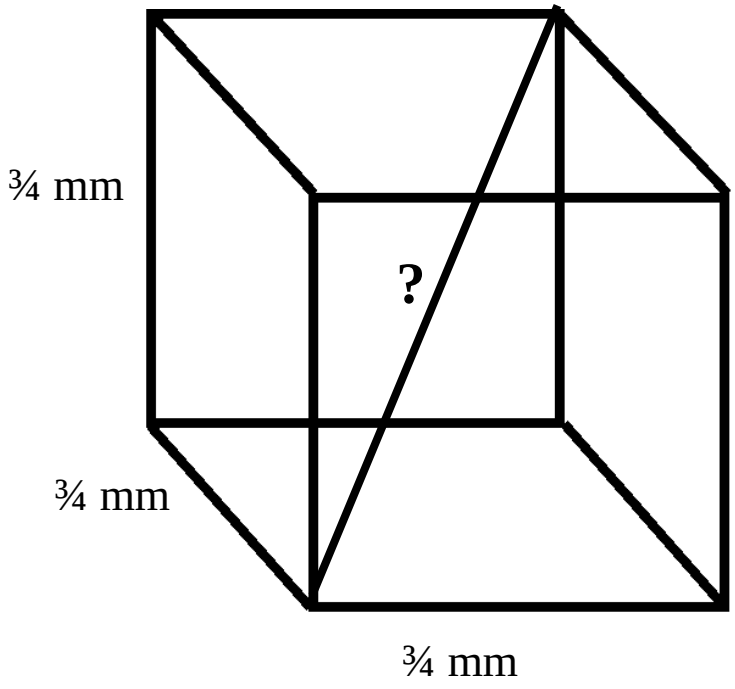
Team Number: _____ School: _____

Problem 3 **2 points** **2 minutes**

A cube with edges ¾ mm (millimeter) long is shown below.

1. What is the **sum of the total length of the <u>edges</u>** of the cube?
2. What is the length, in mm, of a **<u>diagonal</u>** that runs from one corner of the cube to the opposite corner?

*Your answer may be **exact <u>or</u> approximate. If you choose approximate, round your answer to the nearest thousandth place**.*



Answers: _____

_____

Team Number: _____ School: _____

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Problem # 4

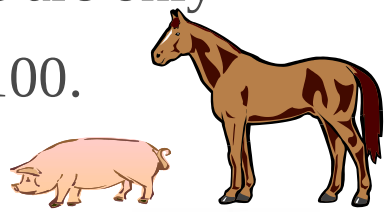
Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 4 **2 points** **2 minutes**

http://www.eklhad.net/funmath.html, #4

Horses cost $10, pigs cost $3, and rabbits are only $0.50. A farmer buys 100 animals for $100. How many of each animal did he buy?



There are two correct answers, find one.

Answer: _____

Team Number: _____ School: _____

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Problem # 5

Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 5 **2 points** **2 minutes**

http://www.eklhad.net/funmath.html, #7

A toy boat floats in a bathtub full of water. The boat contains a rock. A child takes the rock out of the boat and drops it into the water, whereupon the rock promptly sinks to the bottom of the tub.

Is the water level in the tub higher or lower than when the rock was in the boat, or is it the same?

Answer: _____

Team Number: _____ School: _____

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Problem # 6

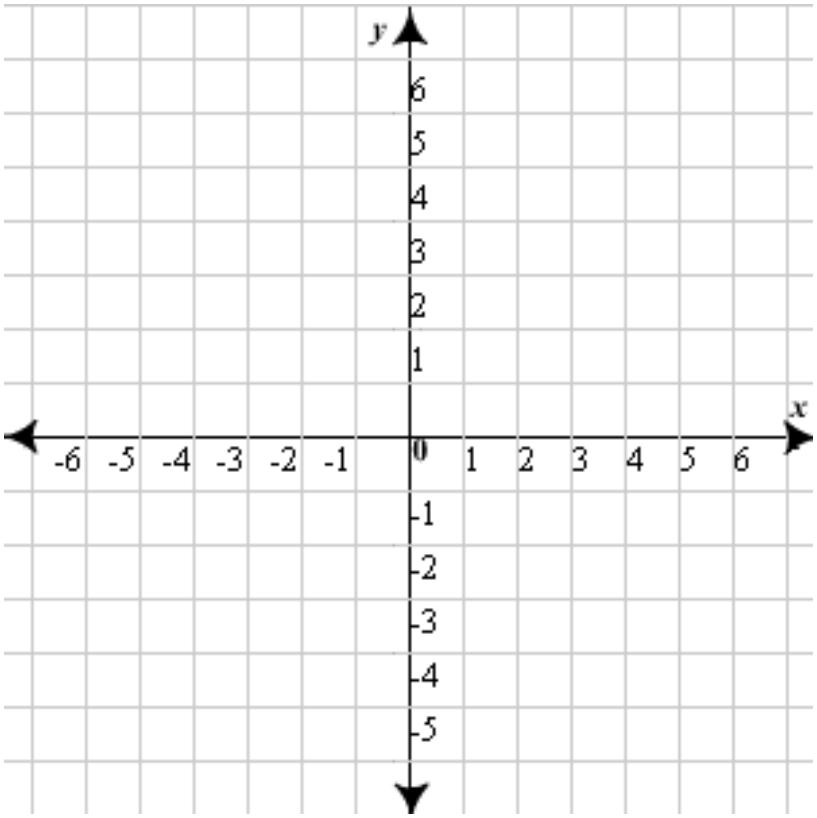
Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 6 **2 points** **2 minutes**

Graph the points (6, 0) and (0,-6). If a line is the **perpendicular bisector** of the line segment joining the endpoints (6, 0) and (0, -6), **what is the equation of the perpendicular bisector** in slope-intercept form?

(Points must be graphed correctly, to be considered a correct answer.)



Answer: _____

Team Number: _____ School: _____

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Problem # 7

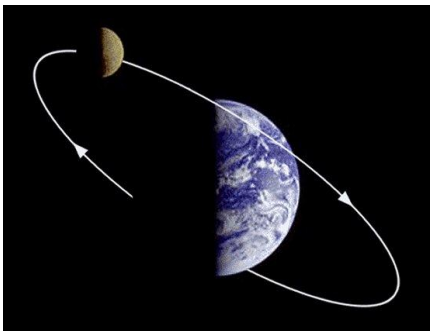
Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 7 **2 points** **2 minutes**

The average radius of the moon's orbit around the earth's center is 61 Earth radii. The radius of Earth is 6371 km. The moon completes an orbit of Earth in approximately 27.5 days. What is **the distance the moon travels around Earth in a year (365 days)?** Round your answer to the nearest whole number.

Use the π button on your calculator for calculations.



Answer: _____

Team Number: _____ School: _____

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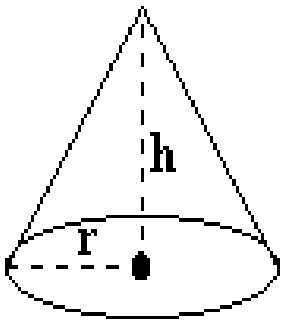
Problem # 8

Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 8 **2 points** **2 minutes**

The diameter and height of a right circular cone are equal.



If the volume of the cone is 3 cubic meters, **what is the height of the cone?** Round your answer to the nearest hundredth of a meter. *Formula:* $V = \frac{1}{3}\pi r^2 h$

Answer: _____

Team Number: _____ **School: _____**

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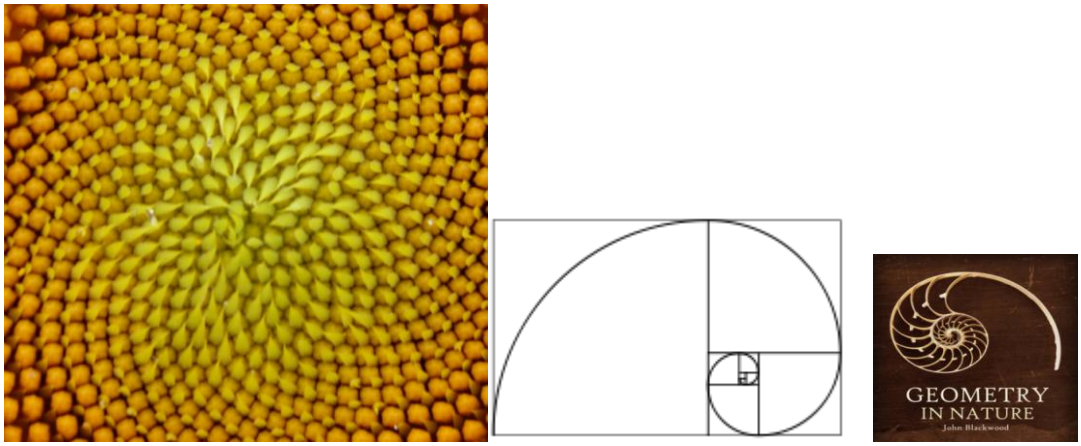
Problem # 9

Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 9 **2 points** **2 minutes**



What are the first 10 numbers in the Fiboncacci sequence?

Answer: _____

Team Number: _____ School: _____

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Problem # 10

Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 10 **1 point** **1 minute**

Modified from http://puzz.freepolls.com/cgi-bin/pollresults/294

You recently returned from a trip to New Zealand. Today is Tuesday. You returned five days before the day after the day before tomorrow. On what day did you return?

Answer: _____

Team Number: _____ **School: _____**

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Problem # 11

Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 11 **2 points** **2 minutes**

http://www.ixl.com/math/grade-8/interpret-stem-and-leaf-plots

Broken tiles per shipment

Stem	Leaf
1	4
2	6
3	2
4	1 1 1 1
5	
6	2 4
7	2 8
8	0 2
9	0

1. How many **shipments** were there?

2. What is the **median** number of broken tiles?

Answers: _____

_____

Team Number: _____ School: _____

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Problem # 12

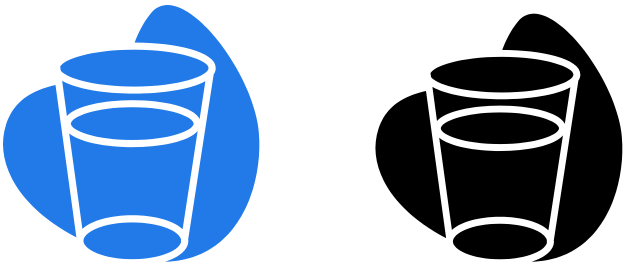
Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 12 **2 points** **2 minutes**

http://www.eklhad.net/funmath.html**, # 8**

You have a glass of water and a glass of juice. Take a spoonful of water from the water glass and add it to the juice. Stir the juice until the water is well mixed throughout. Now take a spoonful of liquid from the juice glass and pour it back into the water glass. **Is there more water in the juice, or more juice in the water, or are the ratios the same?**



Answer: _____

Team Number: _____ School: _____

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Problem # 13

Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 13 **2 points** **2 minutes**

http://www.mathsisfun.com/puzzles/diophantus.html

Diophantus (often known as the 'father of algebra') came from Alexandria and he lived around the year 250 AD.

Diophantus's youth lasted 1/6 of his life. He had the first beard in the next 1/12 of his life. At the end of the following 1/7 of his life Diophantus got married. Five years from then his son was born. His son lived exactly 1/2 of Diophantus's life. Diophantus died 4 years after the death of his son.

How long did Diophantus live?

Answer: _____

Team Number: _____ School: _____

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Problem # 14

Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 14 **2 points** **2 minutes**

Compute the sum of the first hundred numbers in this sequence:

$$16+17+18+...+113+114+115$$

Answer: _____

Team Number: _____ School: _____

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Problem # 15

Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 15 **3 point** **3 minutes**

http://www.fi.edu/school/math/valentine.htm, #9

Match the friends:

Randy has blue eyes.
Matt is 16 and has brown eyes.
Sam is 17 and has green eyes.
Chris has hazel eyes.
Rhonda has green eyes.
Megan is 17 and has blue eyes.
Sabrina is 16 and has hazel eyes.
Carla has brown eyes.

Clues:

No one is friends with someone whose name begins with the same letter.
No one is friends with someone with the same color eyes.
No one is friends with someone the same age.

Answer:

	Rhonda	**Megan**	**Sabrina**	**Carla**
Randy				
Matt				
Sam				
Chris				

Team Number: _____ School: _____

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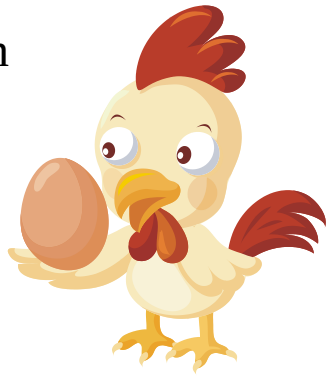
Problem # 16

Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 16 **2 points** **2 minutes**

If a chicken-and-a-half can lay an egg-and-a-half in a day-and-a-half, then how long would it take a pair of chickens to lay a dozen eggs?



Answer: _____

Team Number: _____ **School: _____**

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Problem # 17

Tie Breaker #1

Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 17(Tie Breaker #1) 3 points 3 minutes

There are three Cannibals and three Missionaries on one side of the river with a two passenger boat. Get all six persons to the other side of the river in as few moves as possible. Never let the Cannibals outnumber the Missionaries on either side of the river. They all want to get across the river. No one will wade or swim across.

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Problem # 17

Tie Breaker #2

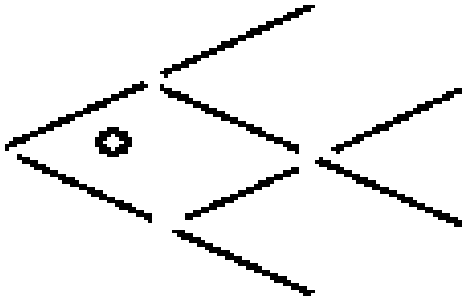
Do NOT turn the page until you are told to do so.

Team Number: _____ School: _____

Problem 18 (Tie Breaker #2) **2 points** **2 minutes**

http://www.fi.edu/sln/school/tfi/spring96/puzzles/puzzle1.html**; #4**

Use 8 toothpicks and 1 button to form a fish:



Move 3 toothpicks and the button to make this fish swim in the opposite direction.

ANSWER KEY

1. There are 6 Triangles depicted in the diagram.
   The difference is 1/3 the total area of Triangle ABC.

2. $\sqrt{3}-\frac{\pi}{2}$ $\sqrt{3}-\frac{\pi}{2}$ $\frac{2\sqrt{3}-\pi}{2}$

   or

   0.156 units^2

3. Friday

   ANSWER: Friday. The day before tomorrow is today - Tuesday. The day after that is Wednesday. Five days before Wednesday is Friday, which is the answer. The original version of this puzzle was created by Terry Stickels.

4. 1.298 or 1.299 mm or

   $\frac{3\sqrt{3}}{4}$

5. First we need to combine the two equations into one. Let h be the number of horses, p the number of pigs, and r the number of rabbits. Double the prices, just for a moment, so we can write 20h + 6p + r = 200. At the same time, h + p + r = 100. Subtract the second from the first to get 19h + 5p = 100. Since h and p are integers, this is a Diophantine equation. There are ways to solve these in general, but in this case we can just try various values of h, to see if p winds up being a whole number. For instance, if h = 0, p = 20, and that works.
   The farmer buys 20 pigs and 80 rabbits.
   The only other combination is 5 horses, 1 pig, and 94 rabbits.

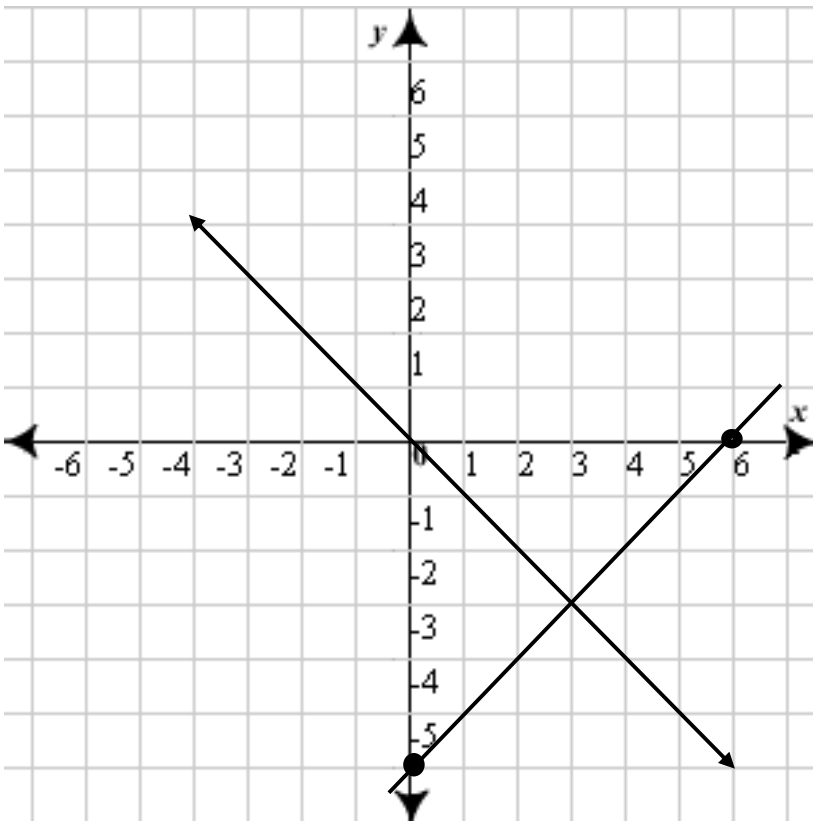
   http://www.eklhad.net/funmath.html, #4

6. In the beginning, the rock caused the water level to rise by an amount that exactly compensates for the weight of the rock. In the end the water level is increased by an amount equal to the volume of the rock. The rock is heavier than water, so its weight displaces more water than its volume. The water level is lower after you throw the rock overboard.

   http://www.eklhad.net/funmath.html, #7

7. Randy-Sabrina; Matt-Rhonda; Sam-Carla; Chris-Megan

8.



y = -x + 0

9. 32,409,884 km (61 * 6371 km * 2 * pi * 365) / 27.5

10. 1.42 meters

11 14 shipments; median = 51.5 broken tiles per shipment

12. **0, 1, 1, 2, 3, 5, 8, 13, 21, 34,** 55

13. *T*here is an equation to reflect the several ages of Diophantus:

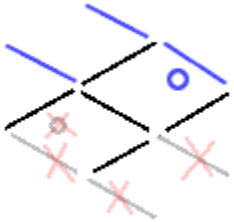
1/6x + 1/12x + 1/7x + 5 + 1/2x + 4 = x

So the solution (x) is 84 years.

http://www.mathsisfun.com/puzzles/diophantus-solution.html

14. At the end, each glass has the same amount of liquid. If there were more water in the juice than juice in the water, there would be more water, in the two glasses taken together, than juice. Yet we started out with equal amounts of water and juice. Both ratios must be the same.

http://www.eklhad.net/funmath.html, #8

15. 

http://www.fi.edu/sln/school/tfi/spring96/puzzles/puzzle1.html; #4

16. 16 + 115 = 131, 17 + 114 = 131, 18 + 113 = 131, 19 + 112 = 131
There are 50 sums of 131 among the consecutive numbers 16 through 115.
Therefore, the total sum is 50 * 131 = 6500 + 50 = 6550

17. 1.5 chickens and lay 1.5 eggs in a day and a half = 1 chicken lays 1 egg in a day and a half. Therefore, 2 chickens (a pair) can lay 2 eggs in a day and a half. So 6 cycles (1.5 days) of 2 eggs each will result in 12 (a dozen) eggs. 6 * 1.5 days = 9 days.
It will take 9 days for a couple of chickens to lay a dozen eggs.

18. A cannibal and missionary go across, and the missionary comes back. Or, two cannibals go across and one comes back.
Two cannibals go across and one comes back.
Two missionaries go across and a missionary and cannibal come back.
Two missionaries go across and the cannibal comes back.
Two cannibals go across and one comes back. Or, one missionary comes back.
Finally the last two cannibals go across, or in the alternate case above, a cannibal and a missionary come back.